**Pin-Level Reconstruction of Various Neutronic Quantities in Fast Reactors: Enhanced Physical Insight and Visualization Tools**

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**INTRODUCTION**

Fast neutrons have long mean free paths. Thus, reactor cores with fast neutron spectra can be modeled on coarse meshes, often with only one mesh cell per hexagonal assembly block. Such coarse meshes yield fairly accurate $k_{eff}$ values and core-wide flux distributions. Unfortunately, they are not well suited for detailed intra-assembly neutronic or thermal hydraulic analysis. While coarse assembly-level meshes are adequate for scoping analyses, build-ready design and regulatory standards require knowledge of neutronic quantities on the pin level. One could take the “brute force” route by solving the neutron diffusion (or transport) equation on a hyperfine pin-level mesh, or one could reconstruct the intra-assembly multigroup flux distributions from coarse-mesh quantities.

Techniques for multigroup flux, power, and burnup reconstruction in fast reactors have been well studied and applied in hexagonal geometry. However, we extend the techniques to reconstruction of other neutronic properties, including current vector fields and adjoint-weighted quantities. We also develop a suite of visualization tools dubbed PyPinPlot (PPP) to flaunt effugent images of reconstructed scalar and vector quantities.

**FLUX AND POWER RECONSTRUCTION**

**Theory**

A coarse assembly-level nodal diffusion solution produces cell-averaged fluxes and surface-averaged partial currents. We follow the procedure outlined by W. S. Yang, et.al. in hexagonal-$z$ geometry. Within each hexagonal prism block, the multigroup (real and adjoint) flux is assumed to be separable in the axial ($z$) and radial ($x,y$) directions. In the radial direction, the reconstructed flux in each energy group preserves six surface-averaged net currents, six surface-averaged fluxes, six corner-point fluxes, and one volume-averaged flux. In the axial direction, the flux reconstruction in each energy group preserves two surface-averaged net currents, two surface-averaged fluxes, and one volume-averaged flux. Thus, there are a total of five axial constraints and nineteen radial constraints. Eq. (1) shows the complete group flux expression; the flux reconstruction process requires computing the $C_{i,g}$ and $C_{j,g}$ coefficients.

$$
\Phi_g(x,y,z) = \left[ \sum_{i=1}^{19} C_{i,g} x^{m_i} y^{n_i} \right] \left[ \sum_{j=1}^{5} C_{j,g} z^{j-1} \right]
$$

We can easily represent the power density as a sum of the same monomials with different weights:

$$
P(x,y,z) = \left[ \sum_{i=1}^{19} C_{i,g} x^{m_i} y^{n_i} \right] \left[ \sum_{j=1}^{5} C_{j,g} z^{j-1} \right]
$$

$$
C_i = \gamma \sum_{a=1}^{g} S_{f,a} C_{i,g}
$$

Here $\gamma$ represents the energy release per fission event. The $C_i$ values are normalized such that the average of the separable $z$ polynomial is 1.

We have benchmarked this reconstruction approach against a hyperfine mesh finite difference diffusion solution for the Fast Flux Test Facility (FFTF). Core-averaged pin power error magnitudes are less than 0.4% for the FFTF benchmark and less than 0.05% for a homogenous core with FFTF fuel.

**Implementation**

We utilize the Advanced Reactor Modeling Interface (ARMI), a Python-based modeling framework that loosely couples nuclear reactor simulations to provide high-fidelity systems analysis. The ARMI neutronics module includes the diffusion solver DIF3D and its embedded transport option VARIANT (VARIational Anisotropic Nodal Transport). We use 33-group cross-section sets generated by MC++2.

We calculate the surface-averaged real and adjoint partial currents via the DIF3D binary output files NHFLUX and NAFLUX, respectively. However, the DIF3D nodal diffusion adjoint solver is unmaintained, so we must employ the VARIANT v10.0 nodal transport adjoint solver to obtain a usable NAFLUX file. The use of VARIANT v10.0 is crucial, as v8.0 will produce an NHFLUX/NAFLUX file format that is exceptionally cumbrous to read.

We develop an ARMI Python module to read surface-averaged partial currents from NHFLUX and/or NAFLUX, generate the 19 radial and 5 axial constraints, and solve matrix equations to produce the $C_{i,g}$ and $C_{j,g}$ values shown in Eq. (1). This module provides pin power...
input to thermal hydraulic, materials, and mechanical analysis. Furthermore, we couple it to a transmutation module to perform pin-level depletion and explore the effects of assembly rotation during shuffling.

We also develop a pin-level plotting tool via the Python graphics package matplotlib. We dub this PyPinPlot (PPP). Fig. 1 shows pin powers in Joyo (a Japanese sodium fast reactor), while Fig. 2 shows the real and adjoint pin powers for a small test core design.

**VECTOR FIELD RECONSTRUCTION: NEUTRON CURRENTS AND POWER GRADIENTS**

Since we have reconstructed an analytical representation for the multigroup flux in each hexagonal block, it is straightforward to obtain an analytical representation for the (real and adjoint) current densities in each block:

\[
\vec{j}_g = -D_g \nabla \Phi_g, \quad \vec{j}_g^\pm = D_g \nabla \Phi_g^\pm
\]

(4)

For example, the \( x \) coordinate of the group flux gradient is

\[
[\nabla \Phi_g]_x = \sum_{j,\delta} C_{j,\delta} m_j x^{m_j-1} y^{m_j} \left[ \sum_{j',\delta'} C_{j',\delta'} z^{j'-1} \right]
\]

(5)

where \( \delta \) is the Dirac delta function. We can plot these currents as vector fields via PPP. Fig. 3 shows an example of the real net neutron current in a small test core. Note that the neutron current field lines bend toward the withdrawn control assemblies.

We can also construct an analytical representation for the power density gradient, assuming that the fission power density (a scalar function) is continuously distributed within each block (and not discretized into pin locations). Fig. 4 exemplifies this.

**ADJOINT-WEIGHTED QUANTITY RECONSTRUCTION**

Since we have reconstructed analytical representations for the real and adjoint fluxes as well as for the real and adjoint flux gradients, it is straightforward (albeit computationally cumbersome) to obtain an analytical representation for the reactivity worths of various materials. The first-order reactivity shift \( \Delta \rho \) due to small perturbations in the standard multigroup diffusion operators \( dF \) and \( dM \) is

\[
\Delta \rho = \frac{\langle \phi^+ | (dF/k - dM) \Phi \rangle}{\langle \phi^+ | F \Phi \rangle}
\]

(6)

If we wish to compute a value that represents the “reactivity worth” of voiding sodium, then we can simply evaluate \( \Phi^+(dM \phi) \). If we already have analytical representations of the flux, the adjoint flux, and their gradients in each block, then an analytical function proportional to the sodium “reactivity worth” per mass is

\[
\phi^+(dM \phi) = \sum_{g=1}^{G} \left( \phi_g^+ \Sigma_{\delta g} \phi_g - (\nabla \phi_g^+ \cdot \nabla \phi_g) \Sigma_{\delta g} \phi_g \right)
\]

(7)

Fig. 5 shows the spatial distribution of this reconstructed “reactivity worth”. It is positive throughout the core except in regions of high flux gradients (high leakage), where it is negative. Sodium worth is also greatly reduced in regions immediately adjacent to withdrawn control rods, where the spectrum tends to be softer.

Also note that in core regions where the real and adjoint currents run parallel (see Fig. 3), neutrons flow toward higher importance. Thus, the leakage component of reactivity worth is actually positive. This means that removing a material actually decreases leakage, because neutrons can more readily flow toward regions of higher importance.

**CONCLUSIONS**

We have extended conventional flux reconstruction techniques to include vector fields and adjoint-weighted quantities. We have also created the aesthetic visualization tool PPP. In particular, the fission power gradient core maps are useful for thermal hydraulic and mechanical analysis, and the sodium worth core maps can potentially elucidate techniques for reducing core-wide void worth. It is not unreasonable to surmise that an intra-assembly analytical representation of any neutronic quantity could be obtained.

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**REFERENCES**

Fig. 1. Pin power reconstruction in the Jōyō sodium fast reactor benchmark displayed via PPP. An inner core of 91-pin fuel assemblies is surrounded by a thick blanket of 19-pin breeder assemblies. The blank hex locations represent withdrawn control assemblies. The image to the right is an enlarged segment of the same core.

Fig. 2. Pin powers in a small 1/3 test core with reflective boundary conditions displayed via PPP. The left image shows the real power $F\Phi$, while the right image shows the “adjoint power” $F^+\Phi$. Note that the real fluxes tend to peak near the withdrawn control assemblies (due to a softened spectrum), while the adjoint fluxes tend to depress in the same areas (reflecting lower neutron importance). In these unique areas, the real and adjoint currents are roughly parallel, while they are roughly antiparallel throughout most of the core.
Fig. 3. The real (top) and adjoint (bottom) net neutron currents in group 5 (1.35 - 2.23 MeV) displayed via PPP for a small 1/3 test core model. Colors represent relative magnitude, while arrows show direction. The real and adjoint currents are nearly always antiparallel, except on the “leeward” side of withdrawn control assemblies. In these unique regions, the real and adjoint currents are roughly parallel, indicating that neutrons flow toward higher importance. Thus, removing material will decrease leakage.

Fig. 4. The fission power gradient displayed via PPP for a small 1/3 test core model. Colors represent relative magnitude, while arrows show direction. The highest power gradients occur on the “leeward” side of withdrawn control assemblies. Such core-wide power gradient maps are elucidating for thermal hydraulic and mechanical analysis, as they often reflect temperature gradients.

Fig. 5. The relative sodium “reactivity worth” distribution displayed via PPP for a small 1/3 test core model. This “reactivity worth” (which is proportional to the local sodium void reactivity coefficient) is positive throughout most of the core, but it is significantly reduced in regions with (1) high leakage and (2) softened spectra. One can see an example of reduced void worth in case (2) surrounding withdrawn control assemblies.